Iowa State University
ECpE Department

EE 455 Introduction to Energy Distribution Systems

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Center-Tapped Transformers and Secondaries

Acknowledgement: The slides are developed based in part on Distribution System Modeling and Analysis, 4th edition, William H. Kersting, CRC Press, 2017

Overview

- The standard method of providing three-wire service to a customer is from a center-tapped single-phase transformer.
- Center-tapped single-phase transformer provides the customer with two 120-V circuits and one 240-V circuit.
- Two types of transformers are available for providing the service to the consumers. (Fig 1 & Fig 2)

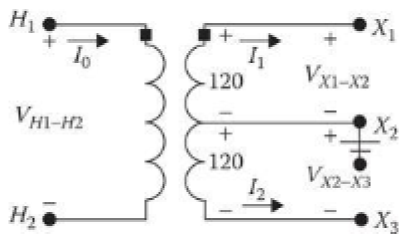


Fig. 1 Center-tapped secondary winding

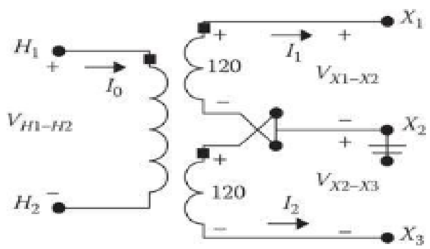


Fig. 2 Three-winding transformer with secondary windings in series

- Fig. 1 has center-tapped winding and provides two 120-V circuits and one 240-V circuit.
- The voltage rating of this transformer is specified as 240/120 V

- Fig. 2 is a three-winding transformer with two secondary windings connected in series.
- The secondary on this transformer is specified as 120/240 V

Overview

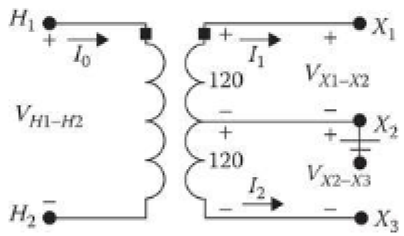


Fig. 1 Center-tapped secondary winding

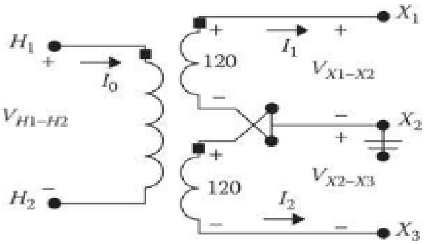


Fig. 2 Three-winding transformer with secondary windings in series

- For both connections, the ideal transformer equations are:

$$n_t = \frac{kVLN_{rated}}{kVLL_{rated}}$$

$$\textbf{example:} \quad n_t = \frac{7200}{240} = 30$$

- $V_{H_1-H_2} = 2 \cdot n_t \cdot Vt_{X_1-X_2}$
- $I_0 = \frac{1}{2 \cdot n_t} \cdot (I_1 - I_2)$

Note: Ideal transformer equations apply to both types of transformers.

Center- Tapped Single-Phase Transformer Equations

GOAL: To derive equation that shows the relationship between Primary Voltage (V_s) and Secondary Voltages (V_1 & V_2) and Currents (I_1 & I_2).

- In Fig. 3, the impedances Z_0, Z_1, and Z_2 represent the individual winding impedances.
- The ideal secondary voltages of the transformer are:

$$\begin{bmatrix} Vt_1 \\ Vt_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} Zt_1 & 0 \\ 0 & -Zt_2 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[Vt_{12}] = [V_{12}] + [Zt_{12}] \cdot [I_{12}] \qquad (1)$$

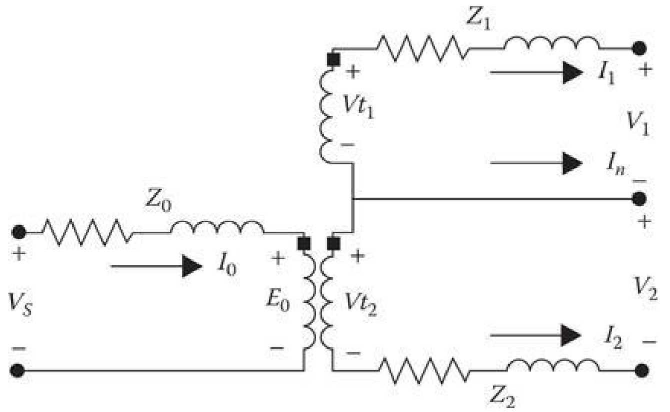


Fig. 3 Center tap transformer model

- The ideal primary voltage as a function of the secondary ideal voltages is:

$$\begin{bmatrix} E_0 \\ E_0 \end{bmatrix} = 2 \cdot n_t \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_{t_1} \\ V_{t_2} \end{bmatrix}$$

$$[E_{00}] = [av] \cdot [V_{t_{12}}] \qquad (2)$$

where,

$$[av] = 2 \cdot n_t \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Substitution of (1) in (2),

$$[E_{00}] = [av] \cdot ([V_{12}] + [Z_{12}] \cdot [I_{12}]) \qquad (3)$$

Center- Tapped Single-Phase Transformer Equations

- The primary transformer current as a function of the secondary winding currents is,

$$I_0 = \frac{1}{2 \cdot n_t} \cdot (I_1 - I_2)$$

$$\begin{bmatrix} I_0 \\ I_0 \end{bmatrix} = \frac{1}{2 \cdot n_t} \cdot \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[I_{00}] = [ai] \cdot [I_{12}] \qquad (4)$$

where,

$$[ai] = \frac{1}{2 \cdot n_t} \cdot \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

The source voltage as a function of the ideal primary voltage is,

$$\begin{bmatrix} V_S \\ V_S \end{bmatrix} = \begin{bmatrix} E_0 \\ E_0 \end{bmatrix} + \begin{bmatrix} Z_0 & 0 \\ 0 & Z_0 \end{bmatrix} \cdot \begin{bmatrix} I_0 \\ I_0 \end{bmatrix}$$

$$[V_{SS}] = [E_{00}] + [Z_{00}] \cdot [I_{00}] \qquad (5)$$

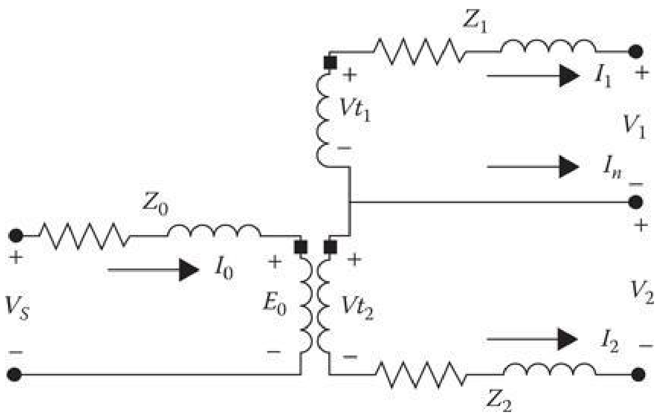


Fig. 3 Center tap transformer model

Center- Tapped Single-Phase Transformer Equations

Substitution of (3) in (5),

$$[V_{ss}] = [av] \cdot [V_{12}] + [av] \cdot [Z_{12}] \cdot [I_{12}] + [Z_{00}] \cdot [I_{00}] \qquad (6)$$

Substitution of (4) in (6),

$$[V_{ss}] = [av] \cdot [V_{12}] + [av] \cdot [Z_{12}] \cdot [I_{12}] + [Z_{00}] \cdot [ai] \cdot [I_{12}]$$
$$[V_{ss}] = [av] \cdot [V_{12}] + ([av] \cdot [Z_{12}] + [Z_{00}] \cdot [ai]) \cdot [I_{12}]$$
$$[V_{ss}] = [a_t] \cdot [V_{12}] + [b_t] \cdot [I_{12}] \qquad (7)$$

where,

$$[a_t] = [av] = 2 \cdot n_t \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[b_t] = [av] \cdot [Z_{12}] + [Z_{00}] \cdot [ai]$$

$$[b_t] = 2 \cdot n_t \cdot \begin{bmatrix} Z_1 + \frac{1}{(2 \cdot n_t)^2} \cdot Z_0 & -\frac{1}{(2 \cdot n_t)^2} \cdot Z_0 \\ \frac{1}{(2 \cdot n_l)^2} \cdot Z_0 & -\left(Z_2 + \frac{1}{(2 - n_t)^2} \cdot Z_0\right) \end{bmatrix}$$

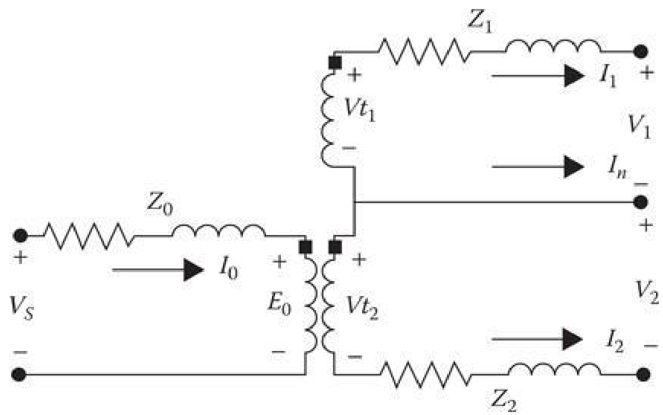


Fig. 3 Center tap transformer model

- Eq. (7) is the backward sweep voltage equation for the single-phase center tapped transformer when the secondary voltages and currents are known.
- Also, Eq. (7) is used to compute the primary source voltage when the secondary terminal voltages and the secondary currents are known.

Center- Tapped Single-Phase Transformer Equations

- To compute the secondary terminal voltages when the primary source voltage and secondary currents are known (Forward Sweep) is derived from Eq. (7).

$$[V_{12}] = [a_t]^{-1} \cdot ([V_{ss}] - [b_t] \cdot [I_{12}])$$

$$[V_{12}] = [a_t]^{-1} \cdot [V_{ss}] - [a_t]^{-1} \cdot [b_t] \cdot [I_{12}]$$

$$[V_{12}] = [A_t] \cdot [V_{ss}] - [B_t] \cdot [I_{12}] \qquad (8)$$

where,

$$[A_t] = [a_t]^{-1} = \frac{1}{2 \cdot n_t} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[B_t] = [a_t]^{-1} \cdot [b_t] = \frac{1}{2 \cdot n_t} \cdot b_t$$

$$[B_t] = \begin{bmatrix} Z_1 + \frac{1}{(2 \cdot n_t)^2} \cdot Z_0 & -\frac{1}{(2 \cdot n_t)^2} \cdot Z_0 \\ \frac{1}{(2 \cdot n_l)^2} \cdot Z_0 & -\left(Z_2 + \frac{1}{(2 - n_t)^2} \cdot Z_0\right) \end{bmatrix}$$

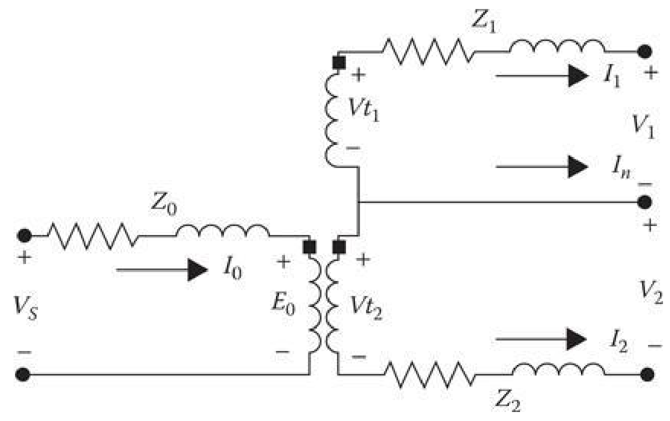


Fig. 3 Center tap transformer model

- Eq. (8) is the forward sweep voltage equation for the single-phase center tapped transformer when the source voltages and secondary currents are known.

Center- Tapped Single-Phase Transformer Equations

Likewise, the primary current as a function of the secondary voltages and currents is given by the backward sweep current equation as,

$$[I_{00}] = [c_t] \cdot [V_{12}] + [d_t] \cdot [I_{12}] \quad (9)$$

where,

$$[c_t] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$[d_t] = \frac{1}{2 - n_t} \cdot \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

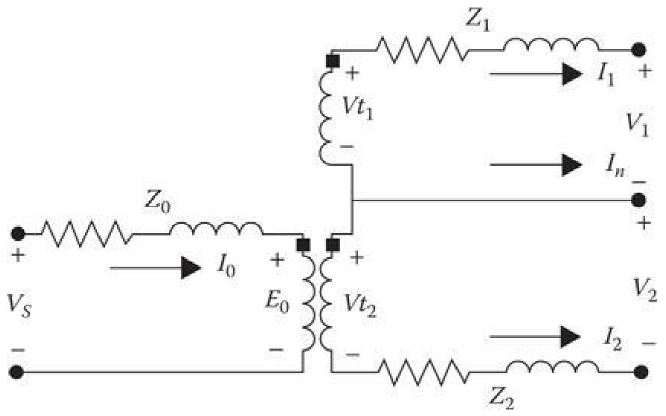


Fig. 3 Center tap transformer model

Center- Tapped Transformers Serving Loads through a Triplex Secondary

- Fig 3. shows a center-tapped transformer serving a load through a triplex secondary.
- Impedance matrix for the triplex secondary needs to be determined before modeling the system
- Impedances of the triplex are computed using Carson's equations and the Kron reduction method.
- Carson's equations result in a 3 × 3 matrix.
- The Kron reduction method is used to "fold" the impedance of the neutral conductor into that of the two-phase conductors.
- A triplex secondary consisting of two insulated conductors and one uninsulated neutral conductor is shown in Figure 11.7
- Spacings between conductors applied in Carson's equations are given as,

$$D_{12} = \frac{\text{dia}+2\cdot T}{12}$$
$$D_{13} = \frac{\text{dia}+T}{12}$$
$$D_{23} = \frac{\text{dia}+T}{12}$$

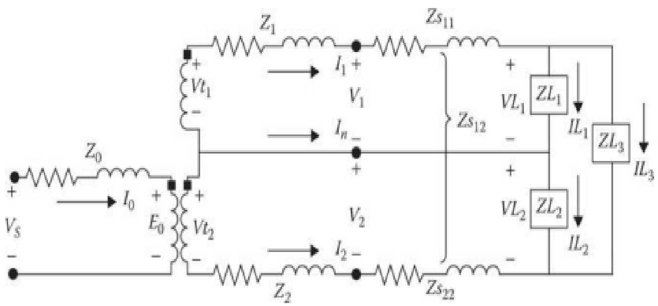


Fig. 4 Center-tapped transformer with secondary

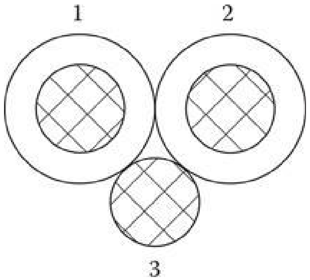


Fig. 5 Triplex Secondary

Center- Tapped Transformers Serving Loads through a Triplex Secondary

- Applying Carston's equations,

$$zp_{ii} = r_i + 0.09530 + j0.12134 \cdot \left(\ln \frac{1}{GMR_t} + 7.93402 \right)$$

$$zp_{ij} = 0.09530 + j0.12134 \cdot \left(\ln \frac{1}{D_{ij}} + 7.93402 \right)$$

where,
r_i = conductor resistance in Ω/ mile
GMR_i = conductor geometric mean radius in ft
D_{ij} = distance in ft between conductors i and j

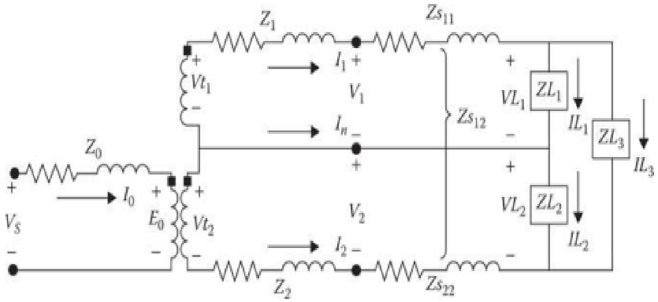


Fig. 4 Center-tapped transformer with secondary

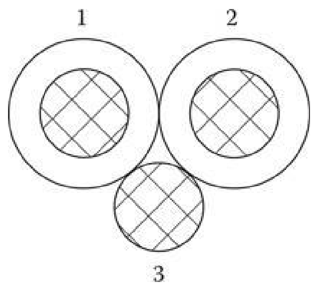


Fig. 5 Triplex Secondary

- The secondary voltage equation in matrix form is:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_n \end{bmatrix} = \begin{bmatrix} V_{1g} \\ V_{2g} \\ V_{ng} \end{bmatrix} - \begin{bmatrix} V_{L_{1g}} \\ V_{L_{2g}} \\ V_{L_{ng}} \end{bmatrix} = \begin{bmatrix} zp_{11} & zp_{12} & zp_{13} \\ zp_{21} & zp_{22} & zp_{23} \\ zp_{31} & zp_{32} & zp_{33} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_n \end{bmatrix}$$

- When the neutral is grounded at the transformer and the load, then:

$$v_n = V_{ng} - V_{L_{ng}} = 0$$

Center- Tapped Transformers Serving Loads through a Triplex Secondary

- This leads to the Kron reduction equation in partitioned form:

$$\begin{bmatrix}[v_{12}]\\ [0]\end{bmatrix} = \begin{bmatrix}[zp_{ii}] & [zp_{in}]\\ [zp_{nj}] & [zp_{nn}]\end{bmatrix} \cdot \begin{bmatrix}[I_{12}]\\ [I_n]\end{bmatrix}$$

Solving for neutral current,

$$[I_n] = -[zp_{nn}]^{-1} \cdot [zp_{ni}] \cdot [I_{12}]$$
$$[I_n] = [t_n] \cdot [I_{12}]$$

where,
$$[t_n] = -[zp_{nn}]^{-1} \cdot [zp_{ni}]$$

The Kron reduction gives the 2×2 phase impedance matrix:

$$[zs] = [zp_{ij}] - [zp_{in}] \cdot [zp_{nn}]^{-1} \cdot [zp_{nj}]$$

- For a secondary of Length 'L',

$$[Zs] = [zs] \cdot L = \begin{bmatrix} Zs_{11} & Zs_{12}\\ Zs_{21} & Zs_{22}\end{bmatrix}$$

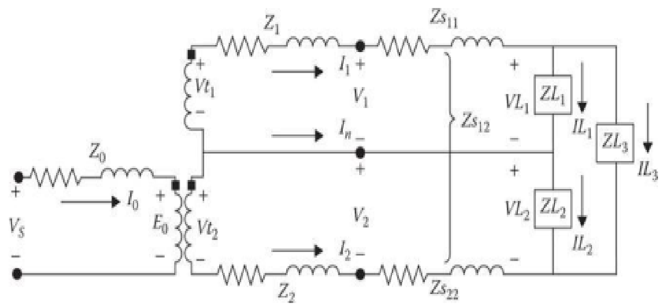


Fig. 4 Center-tapped transformer with secondary

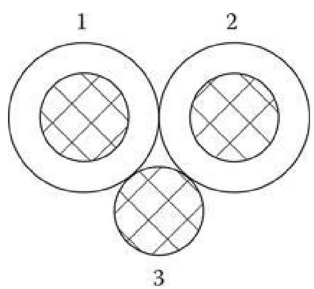


Fig. 5 Triplex Secondary

Center- Tapped Transformers Serving Loads through a Triplex Secondary

From Fig. 4, the voltage backward sweep for the secondary is given by:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} VL_1 \\ VL_2 \end{bmatrix} + \begin{bmatrix} Zs_{11} & Zs_{12} \\ Zs_{21} & Zs_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V_{12}] = [a_{\text{sec}}]VL_{12}] + [b_{\text{sec}}] \cdot [I_{12}] \qquad (9)$$

where,

$$[a_{\text{sec}}] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[b_{\text{sec}}] = \begin{bmatrix} Zs_{11} & Zs_{12} \\ Zs_{21} & Zs_{22} \end{bmatrix}$$

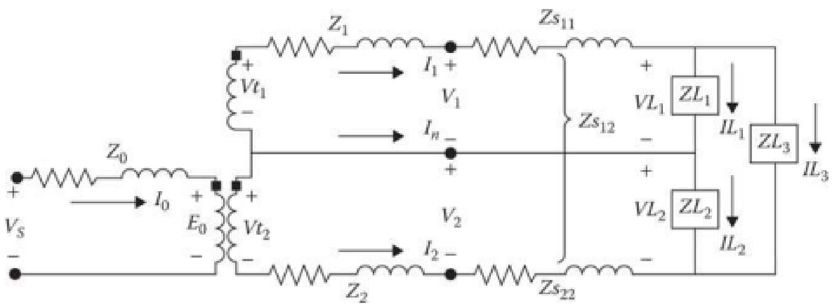


Fig. 4 Center-tapped transformer with secondary

- Because of the short length of the secondary, the line currents leaving the transformer are equal to the line currents at the load.
- Thus, no backward sweep is need.
- However, to remain consistent for the general analysis of a feeder the matrix $[d_{sec}]$ is defined as:

$$[I_{12}] = [d_{\text{sec}}] \cdot [I_{12}] \quad \text{where,} \quad [d_{\text{sec}}] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

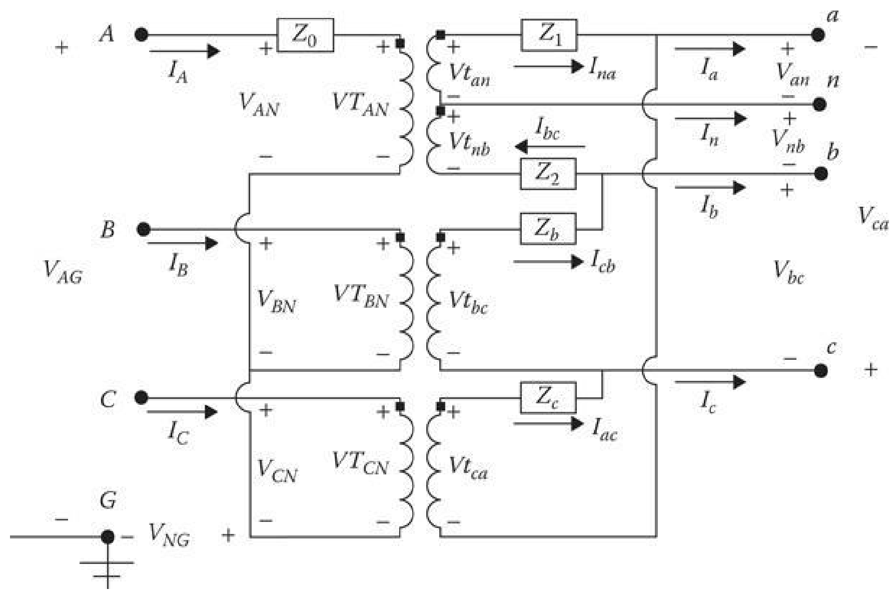


Fig. 5 Ungrounded Wye – Delta Transformer center-tapped transformer connection

- The turns ratio for all transformer are given by:

$$n_t = \frac{kVLN_{rated\ primary}}{kVLL_{rated\ decondary}}$$

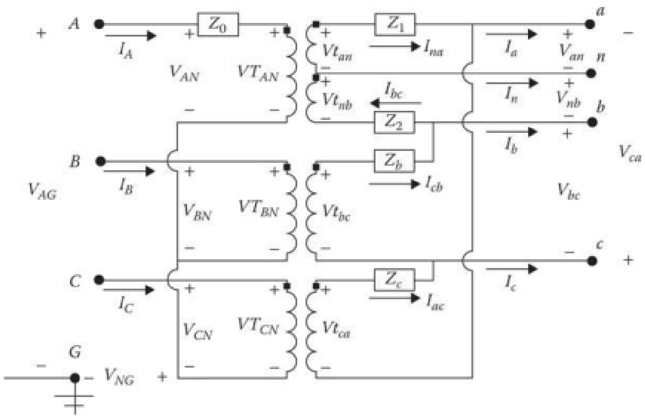
Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

The basic transformer equations for the center tap transformer are:

- $Vt_{an} = Vt_{nb} = \frac{1}{2 \cdot n_t} \cdot VT_{AN}$

  $VT_{AN} = 2 \cdot n_t \cdot Vt_{an}$

- $I_A = \frac{1}{2 \cdot n_t} \cdot (I_{na} + I_{bn})$



For the transformer bank, the basic "ideal" transformer voltage equations as a function of the turn's ratio are:

$$\begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix} = n_t \cdot \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Vt_{an} \\ Vt_{nb} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix}$$

$$[VT_{LN_{ABC}}] = [AV] \cdot [Vt_{anbc}]$$ where, $$[AV] = n_t \cdot \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

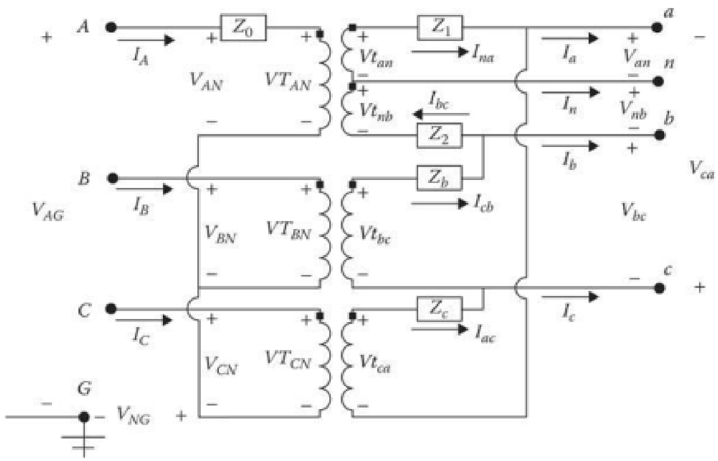
Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

$$\begin{bmatrix} Vt_{an} \\ Vt_{nb} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix}$$

$$[Vt_{anbc}] = [BV] \cdot [VT_{LN_{ABC}}]$$

where,

$$[BV] = \frac{1}{n_t} \cdot \begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



- The basic "ideal" transformer current equations as a function of the turn's ratio are:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{na} \\ I_{bn} \\ I_{cb} \\ I_{ac} \end{bmatrix}$$

where, $[AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$[I_{ABC}] = [AI] \cdot [ID_{anbc}]$$

Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

- **Forward Sweep**

➢ In the forward sweep, the line-to-ground voltages at the terminals of the transformer bank will be known.

$$[V_{LG_{ABC}}] = \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix}$$

➢ In order to determine the voltages across the transformer, it is necessary to first determine the "ideal" primary voltages defined as:

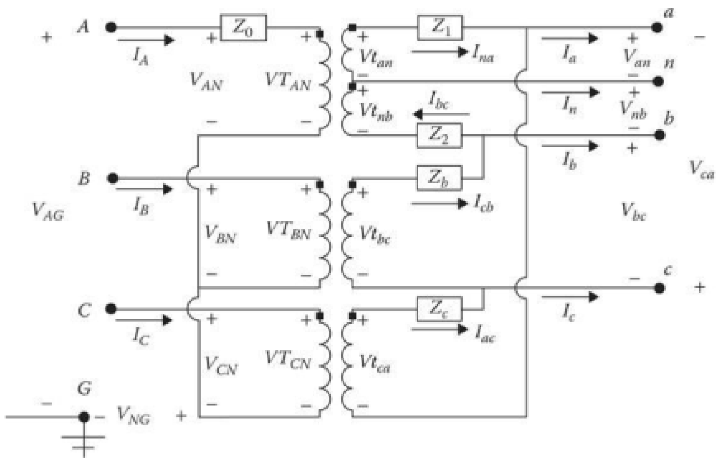
$$[VT_{LN_{ABC}}] = \begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix}$$

➢ The first step is to determine the voltages of the "ideal" transformer to the ground.

$$[VT_{LG_{ABC}}] = [V_{LG_{ABC}}] - [ZT_0] \cdot [I_{ABC}]$$

where,

$$[VT_{LG_{ABC}}] = \begin{bmatrix} VT_{AG} \\ VT_{BG} \\ VT_{CG} \end{bmatrix} \quad , [ZT_0] = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad , [I_{ABC}] = \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$



Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

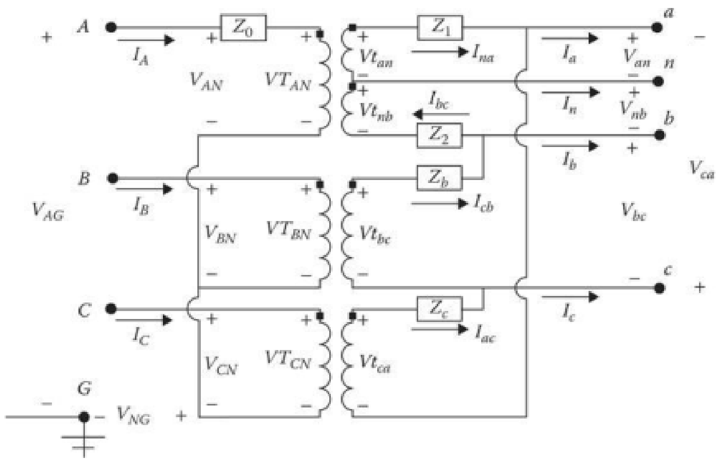
- **Forward Sweep**

➢ The line-to-line "ideal" voltages are:

$$[VT_{LL_{ABC}}] = [Dv] \cdot [VT_{LG_{ABC}}]$$

where,

$$[VT_{LLABC}] = \begin{bmatrix} VT_{AB} \\ VT_{BC} \\ VT_{CA} \end{bmatrix}, [Dv] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$



➢ Primary "ideal" voltages to ground as a function of the primary line-to-ground voltages are:

$$VT_{AN} = VT_{AG} - V_{NG}$$
$$VT_{BN} = VT_{BG} - V_{NG}$$
$$VT_{CN} = VT_{CG} - V_{NG}$$
$$[VT_{LN_{ABC}}] = [VT_{LG_{ABC}}] - [VNG] \qquad (10)$$

where,

$$[VNG] = \begin{bmatrix} V_{NG} \\ V_{NG} \\ V_{NG} \end{bmatrix}$$

➢ The primary line-to-line voltages across the "ideal" transformer windings are:

$$[VT_{LL_{ABC}}] = [Dv] \cdot [VT_{LN_{ABC}}] \qquad (11)$$

Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

- **Forward Sweep**

On substitution Eq. (10) in Eq. (11)

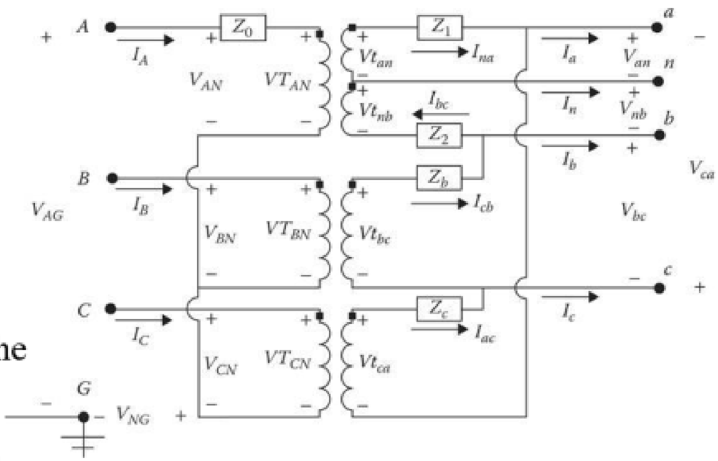
$$[VT_{LL_{ABC}}] = [Dv] \cdot [VT_{LG_{ABC}}] - [Dv] \cdot [VNG] \qquad (12)$$

however,

$$[Dv] \cdot [VNG] = [0]$$

therefore,

$$[VT_{LL_{ABC}}] = [Dv] \cdot [VT_{LG_{ABC}}]$$



- In Eq. 12, the "ideal" line-to-line voltages are known.
- The "ideal" line-to-neutral voltages are needed to continue the forward sweep.
- In Eq. 11, it appears that the line-to neutral voltages can be computed by using the inverse of the matrix $[Dv]$, however the matrix is singular.
- Two of the equations in Eq. (11) can be used, but a third independent equation is needed.
- The two equations from (11) that will be used are:

$$VT_{BC} = VT_{BN} - VT_{CN}$$
$$VT_{CA} = VT_{CN} - VT_{AN} \qquad (13)$$

Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

- **Forward Sweep**

➢ The third independent equation comes from writing Kirchhoff's Voltage Law (KVL) around the delta secondary.

➢ The sum of the secondary voltages around the delta must be equal to zero.

$$V_{an} + V_{nb} + V_{bc} + V_{ca} = 0 \qquad (14)$$

$$Vt_{an} - Z_1 \cdot I_{na} + Vt_{nb} - Z_2 \cdot I_{bn} + Vt_{bc} - Z_b \cdot I_{cb} + Vt_{ca} - Z_c \cdot I_{ac} = 0$$

$$Vt_{an} + Vt_{nb} + Vt_{bc} + Vt_{ca} = Z_1 \cdot I_{na} + Z_2 \cdot I_{bn} + Z_b \cdot I_{cb} + Z_c \cdot I_{ac}$$

$$Vt_{an} + Vt_{nb} + Vt_{bc} + Vt_{ca} = [Z_{D_{anbc}}] \cdot [I_{D_{anbc}}]$$

where,

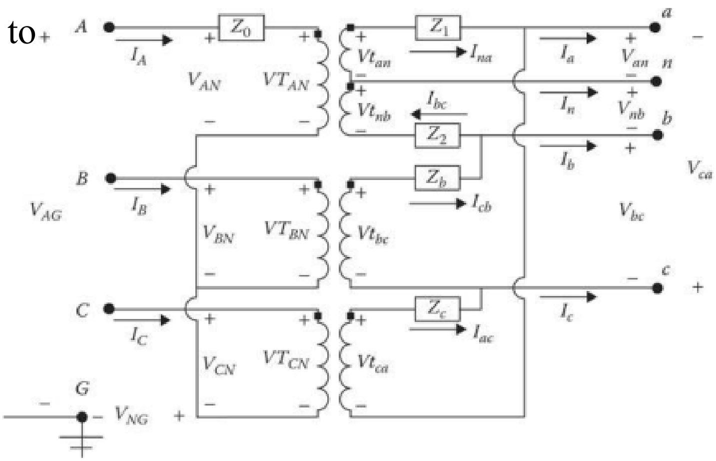
$$[Z_{D_{anbc}}] = [Z_1, Z_2, Z_b, Z_c], \qquad [I_{D_{anbc}}] = \begin{bmatrix} I_{na} \\ I_{bn} \\ I_{cb} \\ I_{ac} \end{bmatrix}$$

but,

$$Vt_{an} + Vt_{nb} + Vt_{bc} + Vt_{ca} = \frac{1}{n_t} \cdot \left(\frac{VT_{AN}}{2} + \frac{VT_{AN}}{2} + VT_{BN} + VT_{CN} \right)$$

$$VT_{AN} + VT_{BN} + VT_{CN} = n_t \cdot [Z_{D_{anbc}}] \cdot [I_{D_{anbc}}] = X$$

where, $X = n_t \cdot [Z_{D_{anbc}}] \cdot [I_{D_{anbc}}]$



Note: The secondary transformer currents will be set to zero in Eq. (14) during the first forward sweep, after that, the most recent secondary current from the backward sweep will be used.

Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

- **Forward Sweep**

➢ Eq.(13) and Eq. (14) are combined in matrix form as:

$$\begin{bmatrix} X \\ VT_{BC} \\ VT_{CA} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix}$$

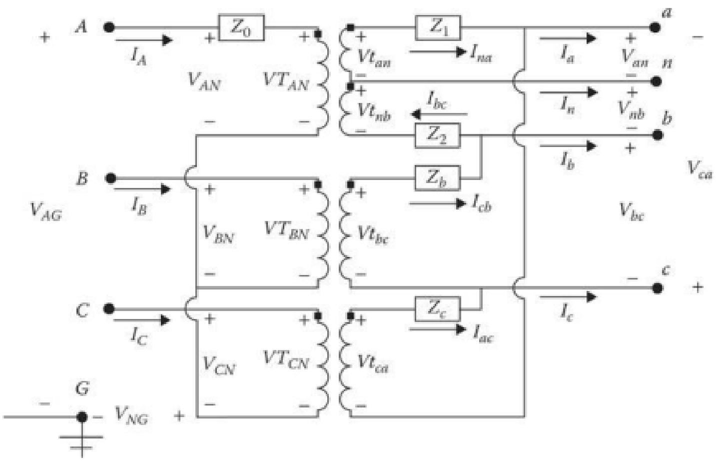
$$[VX_{LL_{ABC}}] = [DX] \cdot [VT_{LN_{ABC}}]$$

➢ The ideal voltages are computed by taking the inverse of $[DX]$.

$$[VT_{LN_{ABC}}] = [Dx] \cdot [VX_{LL_{ABC}}]$$

where,

$$[Dx] = [DX]^{-1} = \frac{1}{3} \cdot \begin{bmatrix} 1 & -1 & -2 \\ 1 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$



➢ With the "ideal" line-to-neutral voltages known, the forward sweep continues with the computation of the secondary "ideal" voltages.

$$\begin{bmatrix} Vt_{an} \\ Vt_{nb} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix} \qquad (15)$$

$$[Vt_{anbc}] = [BV] \cdot [VT_{LN_{ABC}}]$$

Note: The secondary transformer currents will be set to zero in Eq. (14) during the first forward sweep, after that, the most recent secondary current from the backward sweep will be used.

Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

- **Forward Sweep**

➢ The secondary transformer terminal voltages are given by:

$$\begin{bmatrix} V_{an} \\ V_{nb} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} Vt_{an} \\ Vt_{nb} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} - \begin{bmatrix} Z_1 & 0 & 0 & 0 \\ 0 & Z_2 & 0 & 0 \\ 0 & 0 & Z_y & 0 \\ 0 & 0 & 0 & Z_z \end{bmatrix} \cdot \begin{bmatrix} I_{na} \\ I_{bn} \\ I_{cb} \\ I_{ac} \end{bmatrix}$$

$$[V_{anbc}] = [Vt_{anbc}] - [Zt_{anbc}] \cdot [I_{D_{anbc}}]$$

➢ In the first forward sweep, the secondary delta currents are assumed to be zero.

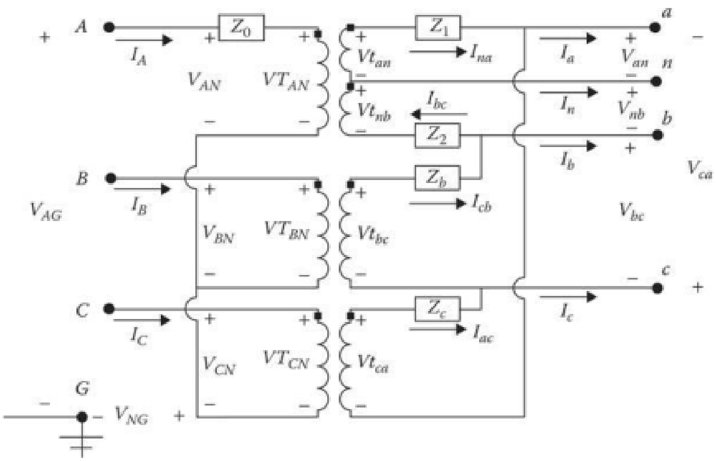
➢ On the first backward sweep, the secondary line currents will be known.

➢ In order to determine the currents in the delta as a function of the line currents, only three Kirchhoff's Current Law (KCL) equations can be used.

➢ The fourth independent equation comes from recognizing that the sum of the primary line currents must be equal to zero.

➢ The three KCL equations to use are:

$$\begin{aligned} I_a &= I_{na} - I_{ac} \\ I_b &= I_{cb} - I_{bn} \\ I_c &= I_{ac} - I_{cb} \end{aligned} \qquad (16)$$



Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

- **Forward Sweep**

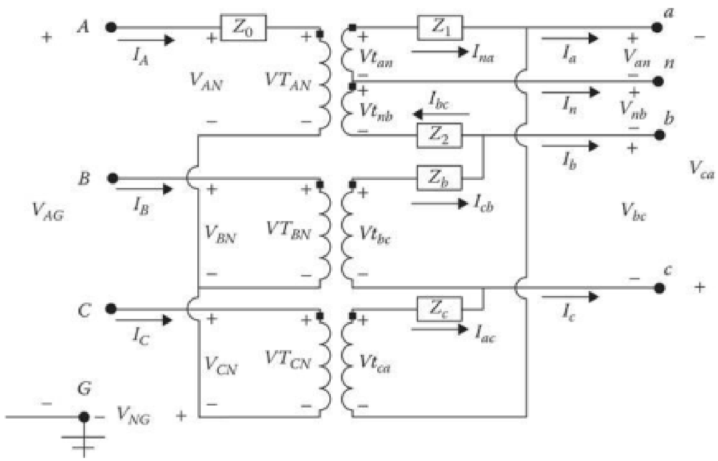
➢ Because the sum of the line currents must equal zero, the fourth equation is given by:

$$I_A + I_B + I_C = 0 = \frac{1}{2 \cdot n_t} \cdot (I_{na} + I_{bn}) + \frac{1}{n_t} \cdot (I_{cb} + I_{ac})$$

$$I_A + I_B + I_C = 0 = \frac{1}{2 \cdot n_t} \cdot (I_{na} + I_{bn} + 2 \cdot I_{cb} + 2 \cdot I_{ac}) \qquad (17)$$

$$2 \cdot n_t \cdot (I_A + I_B + I_C) = 0 = I_{na} + I_{bn} + 2 \cdot I_{cb} + 2 \cdot I_{ac}$$

$$0 = I_{na} + I_{bn} + 2 \cdot I_{cb} + 2 \cdot I_{ac}$$



➢ Combining Eq. (16) and (17) into matrix form:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} I_{na} \\ I_{bn} \\ I_{ca} \\ I_{ac} \end{bmatrix}$$

$$[I_{abc0}] = [X_1] \cdot [I_{D\ anbc}]$$

➢ The delta currents can now be computed by taking the inverse of $[X1]$,

$$[I_{D_{\text{anbc}}}] = [X_1]^{-1} \cdot [I_{abc0}] \qquad (18)$$

$$[I_{\text{Danbc}}] = [x_1] \cdot [I_{abc0}]$$

$$\begin{bmatrix} I_{na} \\ I_{bn} \\ I_{ca} \\ I_{ac} \end{bmatrix} = \frac{1}{6} \cdot \begin{bmatrix} 5 & 1 & 3 & 1 \\ -1 & -5 & -3 & 1 \\ -1 & 1 & -3 & 1 \\ -1 & 1 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \\ 0 \end{bmatrix}$$

$$[I_{D_{anbc}}] = [x1] \cdot [I_{abc0}]$$

where, $[x_1] = [X_1]^{-1} = \frac{1}{6} \cdot \begin{bmatrix} 5 & 1 & 3 & 1 \\ -1 & -5 & -3 & 1 \\ -1 & 1 & -3 & 1 \\ -1 & 1 & 3 & 1 \end{bmatrix}$

Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

- **Forward Sweep**

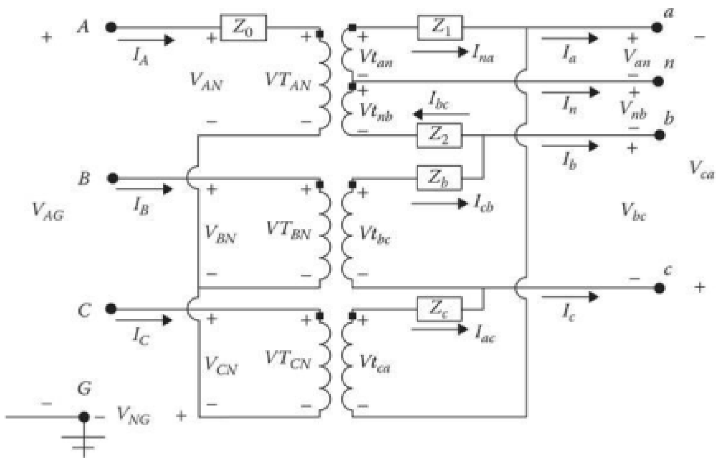
➢ In Eq. 18, the fourth column of the inverse is not needed, as the fourth term in the current vector is zero. The modified Eqn is shown in (19)

$$\begin{bmatrix} I_{na} \\ I_{bn} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \frac{1}{6} \cdot \begin{bmatrix} 5 & 1 & 3 \\ -1 & -5 & -3 \\ -1 & 1 & -3 \\ -1 & 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (19)$$

$$[I_{D_{anbc}}] = [Dd] \cdot [I_{abc}]$$

where,

$$[Dd] = \frac{1}{6} \cdot \begin{bmatrix} 5 & 1 & 3 \\ -1 & -5 & -3 \\ -1 & 1 & -3 \\ -1 & 1 & 3 \end{bmatrix}$$



➢ Note: We have derived following expression in slide 15,

$$[I_{ABC}] = [AI] \cdot [I_{D_{anbc}}]$$

➢ On substitution of Eq. 19 in the above equation, (from slide 15)

$$[I_{ABC}] = [AI] \cdot [Dd] \cdot [I_{abc}] \quad (20)$$

Define:

$$[d_t] = [AI] \cdot [Dd]$$

$$[I_{ABC}] = [d_t] \cdot [I_{abc}]$$

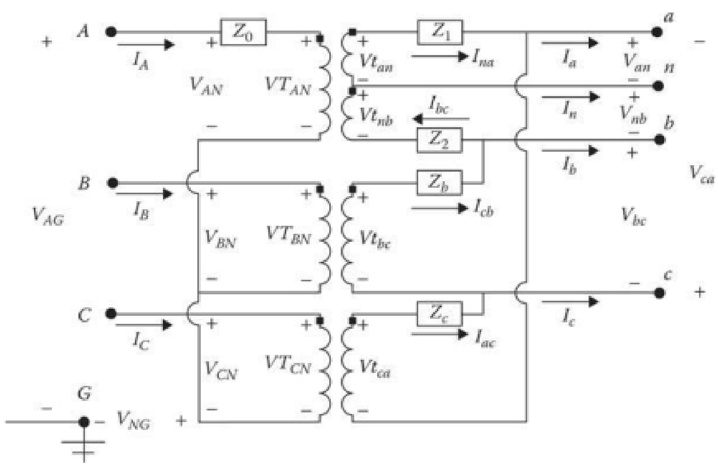
Note: Eqn. (20) is necessary equation used in the backward sweep to compute the primary line currents.

Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

- **Forward Sweep**

➢ With the primary line currents known, the primary line-to-neutral voltages are computed as:

$$[VT_{LN_{ABC}}] = [V_{LN_{ABC}}] - [ZT_0] \cdot [I_{ABC}]$$
$$\text{but: } [I_{ABC}] = [d_t] \cdot [I_{abc}] \qquad (21)$$
$$[VT_{LN_{ABC}}] = [V_{LN_{ABC}}] - [ZT_0] \cdot [d_t] \cdot [I_{abc}]$$



➢ The secondary transformer voltages are computed by:

$$[V_{anbc}] = [Vt_{anbc}] - [Zt_{anbc}] \cdot [I_{D_{anbc}}] \qquad (22)$$
$$[I_{D_{anbc}}] = [Dd] \cdot [I_{abc}]$$
$$[V_{anbc}] = [Vt_{anbc}] - [Zt_{anbc}] \cdot [Dd] \cdot [I_{abc}]$$

➢ Substitute Eqn. (15) into Eqn. (22):

$$[V_{anbc}] = [Vt_{anbc}] - [Zt_{anbc}] \cdot [Dd] \cdot [I_{abc}]$$
$$[Vt_{anbc}] = [BV] \cdot [VT_{LN_{ABC}}] \qquad (23)$$
$$[V_{anbc}] = [BV] \cdot [VT_{LN_{ABC}}] - [Zt_{anbc}] \cdot [Dd] \cdot [I_{abc}]$$

➢ Substitute Eqn. (21) into Eqn. (23):

$$[V_{anbc}] = [BV] \cdot [VT_{LN_{ABC}}] - [Zt_{anbc}] \cdot [Dd] \cdot [I_{abc}]$$

$$[VT_{LN_{ABC}}] = [V_{LN_{ABC}}] - [ZT_0] \cdot [d_t] \cdot [I_{abc}]$$

Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

- **Forward Sweep**

$$[V_{anbc}] = [BV] \cdot ([V_{LN_{ABC}}] - [ZT_0] \cdot [d_t] \cdot [I_{abc}]) - [Zt_{anbc}] \cdot [Dd] \cdot [I_{abc}]$$

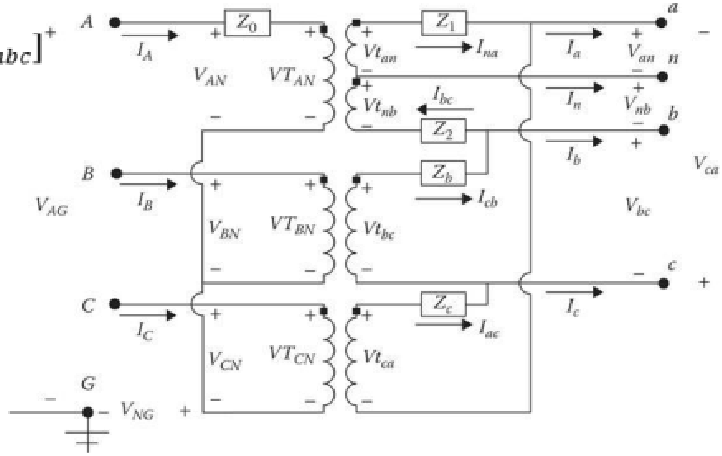
$$[V_{anbc}] = [BV] \cdot [V_{LN_{ABC}}] - ([BV] \cdot [ZT_0] \cdot [d_t] + [Zt_{anbc}] \cdot [Dd]) \cdot [I_{abc}]$$

define:

$$[A_t] = [BV]$$

$$[B_t] = [BV] \cdot [ZT_0] \cdot [d_t] + [Zt_{anbc}] \cdot [Dd]$$

$$[V_{anbc}] = [A_t] \cdot [V_{LN_{ABC}}] - [B_t] \cdot [I_{abc}]$$



Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

- **Backward Sweep**

  - The terminal line-to-neutral voltages are:

$$[V_{LN_{ABC}}] = [VT_{LN_{ABC}}] + [ZT_0] \cdot [I_{ABC}]$$

$$\text{but: } [VT_{LN_{ABC}}] = [AV] \cdot [Vt_{\text{anbc}}] \qquad (24)$$

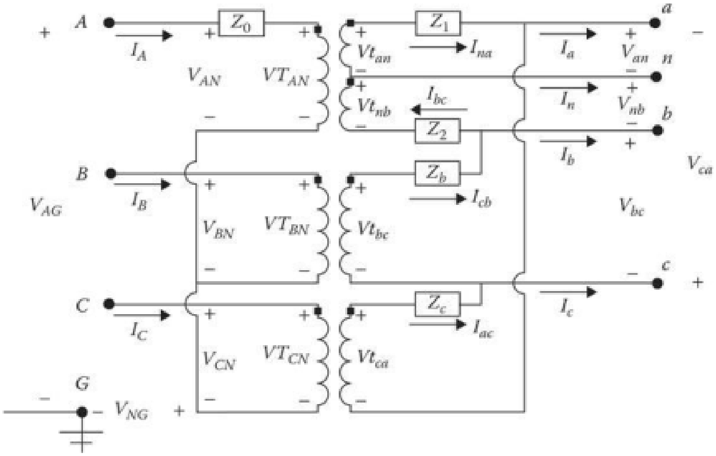
$$\text{and: } [I_{ABC}] = [d_t] \cdot [I_{abc}]$$

$$\text{therefore: } [V_{LN_{ABC}}] = [AV] \cdot [Vt_{\text{anbc}}] + [ZT_0] \cdot [d_t] \cdot [I_{abc}]$$

  - The "ideal" secondary voltages as a function of the secondary terminal voltages are:

$$[Vt_{anbc}] = [V_{anbc}] + [Zt_{anbc}] \cdot [I_{D_{abc}}]$$

$$[Vt_{anbc}] = [V_{anbc}] + [Zt_{anbc}] \cdot [Dd] \cdot [I_{abc}] \qquad (25)$$



  - Substitute Eqn. (25) in Eqn. in (24)

$$[V_{LN_{ABC}}] = [AV] \cdot ([V_{anbc}] + [Zt_{anbc}] \cdot [Dd] \cdot [I_{abc}]) + [d_t] \cdot [I_{abc}]$$

$$[V_{LN_{ABC}}] = [AV] \cdot [V_{anbc}] + ([AV] \cdot [Zt_{anbc}] \cdot [Dd] + [ZT_0] \cdot [d_t]) \cdot [I_{abc}] \qquad (26)$$

therefore:

$$[V_{LN_{ABC}}] = [a_t] \cdot [V_{anbc}] + [b_t] \cdot [I_{abc}] \quad \text{where, } [a_t] = [AV]$$

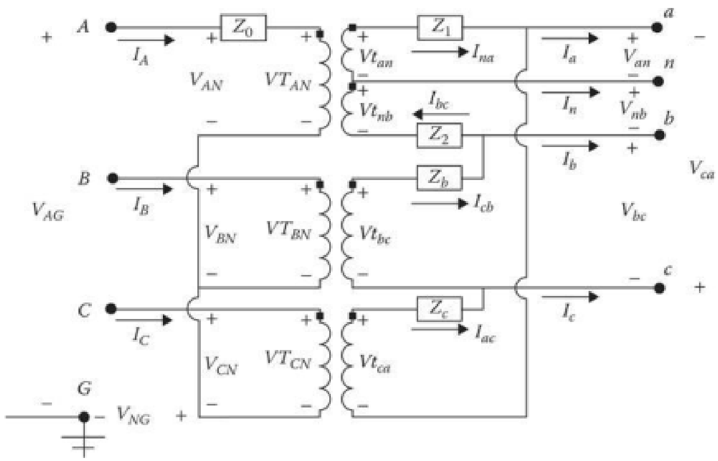
$$[b_t] = [AV] \cdot [Zt_{anbc}] \cdot [Dd] + [ZT_0] \cdot [d_t]$$

Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

- **Backward Sweep**

<u>Note:</u>

- Normally on the backward sweep the node voltages are not computed using Eqn. (26).
- Only the currents are calculated back to the source.
- However, as a check to confirm the final results of the power-flow program, Eqn. (26) using the computed secondary voltages and currents is used to confirm that the source voltages are the same as that which were used in the forward sweep



Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

- **Summary of the Equation Derived:**
  - It is important to note that the turn's ratio is given by:

$$n_t = \frac{kVLN_{hi}}{kV_{LL_{lo}}}$$

  - In the derivation of the forward and backward matrices, it was found that all of the matrices can be defined by the combination of matrices based upon basic circuit theory. The definitions are as follows:

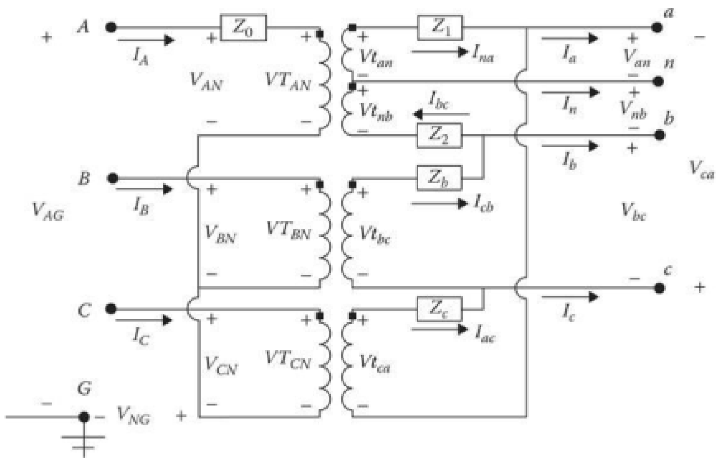
$$[a_t] = [AV]$$

$$[b_t] = [AV] \cdot [Zt_{anbc}] \cdot [Dd] + [ZT_0] \cdot [d_t]$$

$$[d_t] = [AI] \cdot [Dd] \qquad (26)$$

$$[A_t] = [BV]$$

$$[B_t] = [BV] \cdot [ZT_0] \cdot [d_t] + [Zt_{anbc}] \cdot [Dd]$$

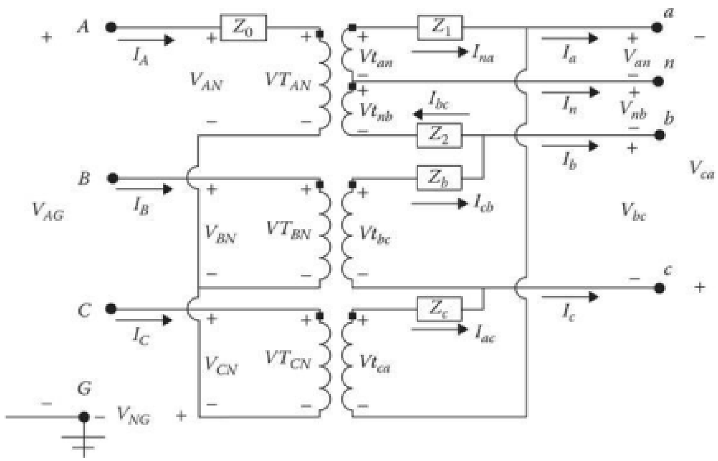


Ungrounded Wye – Delta Transformer Bank with Center-Tapped Transformer

- **Summary of the Equation Derived:**
  - The individual matrices in Eqn. (26) define the relationship between parameters by:

$$\begin{aligned}
\left[I_{D_{anbc}}\right] &= [Dd]\cdot[I_{abc}] \\
\left[VT_{LN_{ABC}}\right] &= [AV]\cdot[Vt_{anbc}] \\
[I_{ABC}] &= [AI]\cdot\left[I_{D_{anbc}}\right] \\
[Vt_{anbc}] &= [BV]\cdot[VT_{ABC}]
\end{aligned} \quad (27)$$

The definitions of Equations (26) and (27) will be used to develop the models for the open wye-open delta connections.



Open Wye - Open Delta Transformer Connections

- Usually, it consists of one lighting transformer and one power transformer.
- The combination of the transformer is used to serve a combination single-phase and three-phase loads.
- For this connection, the neutral of the primary wye-connected windings must be grounded.
- There are two type of connections for an open Wye – Open Delta Transformer Connection.
  a) Leading Open Wye – Open Delta Connection
  b) Lagging Open Wye – Open Delta Connection

Open Wye - Open Delta Transformer Connections

The "leading" Connection

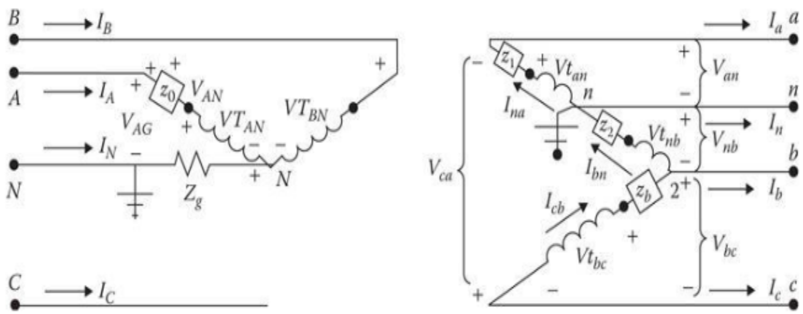


Fig. (a) Leading Open Wye – Open Delta Connection

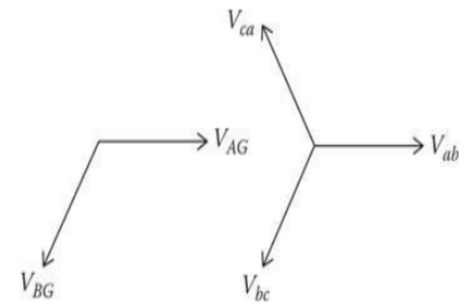


Fig. (b) Leading Open Wye – Open Delta voltage phasors

- The "leading" connection, the voltage applied to the lighting transformer will lead the voltage applied to the power transformer by 120°.
- The voltage phasors at no-load for the leading connection in Fig. (a) are shown in Fig. (b).
- In Fig. (b) there are three line-to-line voltages; two of those voltages come directly from the primary voltages applied to the lighting and power transformers, and the third voltage is the result of line-to-line voltages and must equal to zero.

Open Wye - Open Delta Transformer Connections

<u>The "lagging" Connection</u>

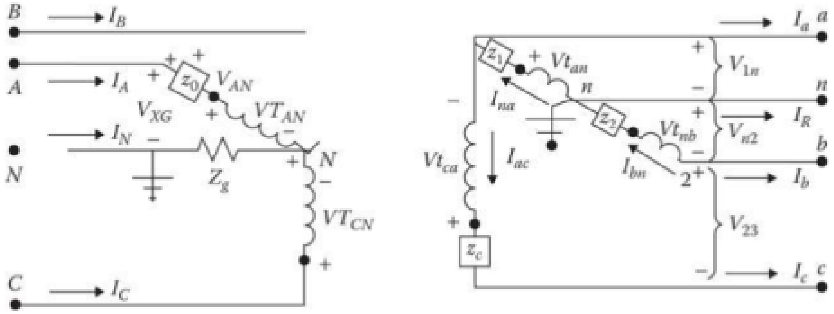


Fig. (a) Lagging open wye – open delta connection

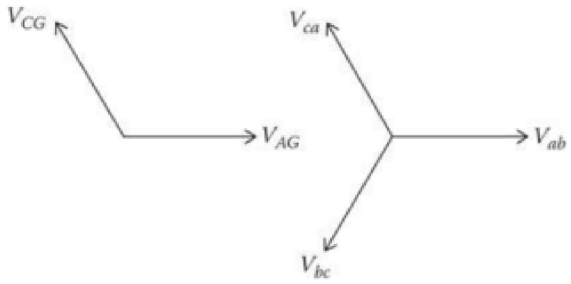


Fig. (b) Lagging Open Wye – Open Delta voltage phasors

- The "lagging" connection, the voltage applied to the lighting transformer will lag the voltage applied to the power transformer by 120°.
- The voltage phasors at no-load for the lagging connection in Fig. (a) are shown in Fig. (b).

Open Wye - Open Delta Transformer Connections Equations

Forward Sweep

- There will be a slight difference in the matrices for the leading and lagging connections.
- In order to define the matrices, the subscript L will be used on various matrices.
  a) L = 1 (leading connection)
  b) L = 2 (lagging connection)
- The "ideal" primary transformer voltages for both connections are:

$$\begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix} = \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} - \begin{bmatrix} Z_0 + Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \quad (27)$$

$$[VT_{LN_{ABC}}] = [VT_{LG_{ABC}}] - [ZOG] \cdot [I_{ABC}]$$

where,

$$[ZOG] = \begin{bmatrix} Z_0 + Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \end{bmatrix}$$

Open Wye - Open Delta Transformer Connections Equations

Forward Sweep

- The "ideal" secondary transformer voltages are:

$$[Vt_{anbc}]_L = [BV]_L \cdot [VT_{LN_{ABC}}]$$

- Leading Connection

$$[Vt_{anbc}]_1 = \begin{bmatrix} Vt_{an} \\ Vt_{nb} \\ Vt_{ab} \end{bmatrix} [BV]_1 = \frac{1}{n_t} \cdot \begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (28)$$

- Lagging Connection

$$[Vt_{anbc}]_2 = \begin{bmatrix} Vt_{an} \\ Vt_{nb} \\ Vt_{ab} \end{bmatrix} [BV]_2 = \frac{1}{n_t} \cdot \begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

- Substitute Eqn. (27) in Eqn. (28),

$$\begin{aligned} [Vt_{anbc}]_L &= [BV]_L \cdot ([V_{LG_{ABC}}] - [Z0G] \cdot [I_{ABC}]) \\ [Vt_{anbc}]_L &= [BV]_L \cdot [V_{LG_{ABC}}] - [BV]_L \cdot [Z0G] \cdot [I_{ABC}] \end{aligned} \quad (29)$$

- The transformer secondary currents are defined as:

$$\text{Leading connection: } [I_{D_{anbc}}]_1 = \begin{bmatrix} I_{na} \\ I_{bn} \\ I_{cb} \end{bmatrix} \qquad \text{Lagging connection: } [I_{D_{anbc}}]_2 = \begin{bmatrix} I_{na} \\ I_{bn} \\ I_{ac} \end{bmatrix} \quad (30)$$

Open Wye - Open Delta Transformer Connections Equations

Forward Sweep

- The primary line currents as a function of the secondary open delta currents are:

$$[I_{ABC}] = [AI]_L \cdot \left[I_{D_{anbc}}\right]_L \qquad (31)$$

where,

$$[AI]_1 = \frac{1}{n_t} \cdot \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[AI]_2 = \frac{1}{n_t} \cdot \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- The secondary open delta currents as a function of the secondary line currents are:

$$\left[I_{D_{anbc}}\right]_L = [Dd]_L \cdot [I_{abcn}] \qquad (32)$$

where,

$$[I_{abcn}] = \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix}, \quad [Dd]_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \quad [Dd]_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Open Wye - Open Delta Transformer Connections Equations

Forward Sweep

- Substitute Equations (31) and (32) in Eqn. (29) :

$$\left[I_{D_{anbc}}\right]_L = [Dd]_L \cdot [I_{abcn}] \quad (33)$$

$$[I_{ABC}] = [AI]_L \cdot \left[I_{D_{anbc}}\right]_L$$

$$[Vt_{anbc}]_L = [BV]_L \cdot \left[V_{LG_{ABC}}\right] - [BV]_L \cdot [Z0G] \cdot [I_{ABC}]$$

$$[Vt_{anbc}]_L = [BV]_L \cdot \left[V_{LG_{ABC}}\right] - [BV]_L \cdot [Z0G] \cdot [AI]_L \cdot [Dd]_L \cdot [I_{abcn}]$$

- The transformer bank secondary voltages are:

$$[V_{abc}]_L = [Vt_{anbc}]_L - [Zt_{\mathrm{sec}}] \cdot \left[I_{D_{anbc}}\right]_L$$

where,

$$[V_{abc}]_1 = \begin{bmatrix} V_{an} \\ V_{nb} \\ V_{bc} \end{bmatrix}, \quad [V_{abc}]_2 = \begin{bmatrix} V_{an} \\ V_{nb} \\ V_{ca} \end{bmatrix} \quad (34)$$

but,

$$\left[I_{D_{anbc}}\right]_L = [Dd]_L \cdot [I_{abcn}]$$

$$[V_{abc}]_L = [Vt_{anbc}]_L - [Zt_{\mathrm{sec}}] \cdot [Dd]_L \cdot [I_{abcn}]$$

Open Wye - Open Delta Transformer Connections Equations

<u>Forward Sweep</u>

- Substitute Equations (33) in Eqn. (34) :

$$[Vt_{anbc}]_L = [BV]_L \cdot [V_{LG_{ABC}}] - [BV]_L \cdot [Z0G] \cdot [AI]_L \cdot [Dd]_L \cdot [I_{abcn}] \qquad (35)$$

$$[V_{abc}]_L = [Vt_{anbc}]_L - [Zt_{anbc}] \cdot [Dd]_L \cdot [I_{abcn}]$$

$$[V_{abc}]_L = \left([BV]_L \cdot [V_{LG_{ABC}}] - ([BV]_L \cdot [Z0G] \cdot [AI]_L . [Dd]_L \cdot [I_{abcn}]\right) - [Zt_{anbc}] \cdot [Dd]_L \cdot [I_{abcn}]$$

$$[V_{abc}]_L = [BV]_L \cdot [V_{LG_{ABC}}] - ([BV]_L \cdot [Z0G] \cdot [AI]_L + [Zt_{anbc}]) . [Dd]_L \cdot [I_{abcn}]$$

- The secondary line voltages are:

$$\begin{bmatrix} V_{an} \\ V_{nb} \\ V_{bc} \\ V_{ca} \end{bmatrix} = [V_{anbc}] = [CV]_L \cdot [V_{abc}]_L \qquad (36)$$

where,

$$[CV]_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$[CV]_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Open Wye - Open Delta Transformer Connections Equations

Forward Sweep

- Substitute Equations (35) in Eqn. (36) :

$$[V_{abc}]_L = [BV]_L \cdot \left[V_{LG_{ABC}}\right] - ([BV]_L \cdot [Z0G] \cdot [AI]_L + [Zt_{anbc}]) \cdot [Dd_L] \cdot [I_{abcn}] \tag{37}$$

$$[V_{anbc}] = [CV]_L \cdot [V_{abc}]_L$$

$$[V_{anbc}] = [CV]_L \cdot [BV]_L \cdot \left[V_{LG_{ABC}}\right] - [CV]_L([BV]_L \cdot [Z0G] \cdot [AI]_L + [Zt_{anbc}]) \cdot [Dd_L] \cdot [I_{abcn}]$$

Define: $[A_t]_L = [CV]_L \cdot [BV]_L$

$$[B_t]_L = [CV]_L([BV]_L \cdot [Z0G] \cdot [AI]_L + [Zt_{anbc}]) \cdot [Dd_L]$$

therefore: $[V_{anbc}] = [A_t]_L \cdot \left[V_{LG_{ABC}}\right] - [B_t]_L \cdot [I_{abcn}]$

Open Wye - Open Delta Transformer Connections Equations

<u>Backward Sweep</u>

- Substitute Equations (32) in Eqn. (33) :

$$\left[I_{D_{\text{anbc}}}\right]_L = [Dd]_L \cdot [I_{abcn}] \tag{37}$$

$$[I_{ABC}] = [AI]_L \cdot \left[I_{D_{\text{anbe}}}\right]_L$$

$$[I_{ABC}] = [AI]_L \cdot [Dd]_L \cdot [I_{abcn}]$$

$$[I_{ABC}] = [d_t]_L \cdot [I_{abcn}]$$

where,

$$[d_t]_L = [AI]_L \cdot [Dd]_L$$

Four Wire Secondary

- The combination of single-phase and three-phase loads will be connected through a length of open four-wire secondary or a quadraplex cable secondary.

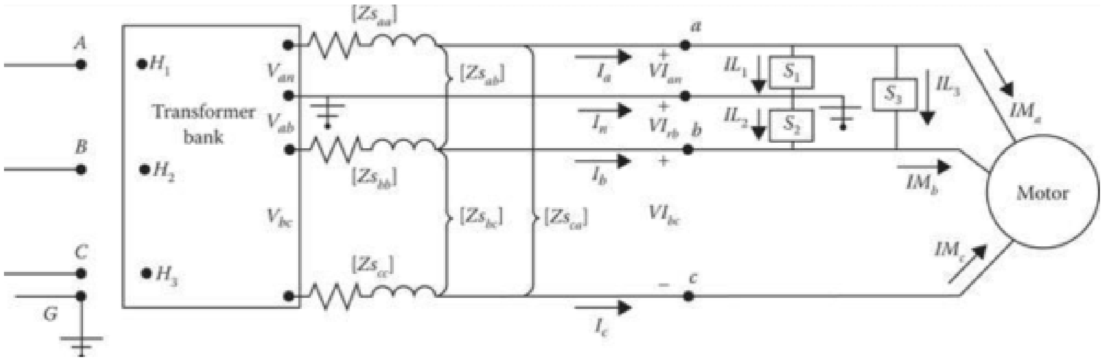


Fig. Four-Wire secondary serving combination loads

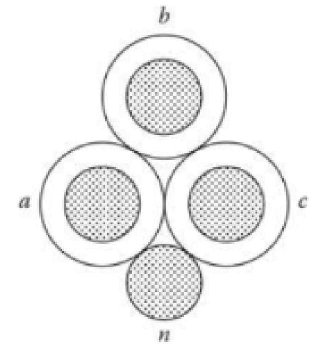


Fig. Quadraplex Cable

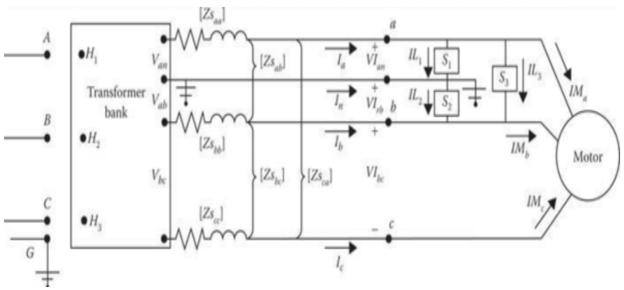
<u>For Modelling four wire secondary,</u>

- The first step is to compute self- and mutual impedances for modeling the open four-wire or quadraplex cable secondary.
- Carson's equations are used to calculate the 4x4 primitive impedance matrix.
- The Kron reduction method is applied to eliminate the fourth row and column since the secondary neutral is grounded at both ends.
- The result is the 3x3 phase impedance matrix.
- Chapter 4 provides more details on how to apply Carson's equations and the Kron reduction.

Modeling of Four Wire Secondary

- The 3x3 phase impedance matrix provides self-impedance for the three line conductors and mutual impedance between them.

$$\begin{aligned} v_a &= Z_{s_{aa}} \cdot I_a + Z_{s_{ab}} \cdot I_b + Z_{s_{ac}} \cdot I_c \\ v_b &= Z_{s_{ba}} \cdot I_a + Z_{s_{bb}} \cdot I_b + Z_{s_{bc}} \cdot I_c \\ v_c &= Z_{s_{ca}} \cdot I_a + Z_{s_{db}} \cdot I_b + Z_{s_{cc}} \cdot I_c \end{aligned} \qquad (38)$$



- The model of the four-wire secondary will again be in terms of the *abcd* and *AB* generalized matrices.

- The first step in developing the model is to write KVL around the three "window" loops and the outside loop in Fig. shown in the right.

$$\begin{aligned} V_{an} &= V_{L_{an}} + v_a \\ V_{nb} &= V_{L_{nb}} - v_b \\ V_{bc} &= V_{L_{bc}} + v_b - v_c \\ V_{ca} &= V_{L_{ca}} + v_c - v_a \end{aligned} \qquad (39)$$

Substitute Eq. (38) in Eqn. (39),

$$\begin{bmatrix} V_{an} \\ V_{nb} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} V_{L_{an}} \\ V_{L_{nb}} \\ V_{L_{bc}} \\ V_{L_{ca}} \end{bmatrix} + \begin{bmatrix} Z_{s_{aa}} & Z_{s_{ab}} & Z_{s_{ac}} \\ -Z_{s_{ba}} & -Z_{s_{bb}} & -Z_{s_{bc}} \\ Z_{s_{ba}} - Z_{s_{ca}} & Z_{s_{bb}} - Z_{s_{db}} & Z_{s_{bc}} - Z_{s_{cc}} \\ Z_{s_{ca}} - Z_{s_{aa}} & Z_{s_d} - Z_{s_{ab}} & Z_{s_{cc}} - Z_{s_{ac}} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \qquad (40)$$

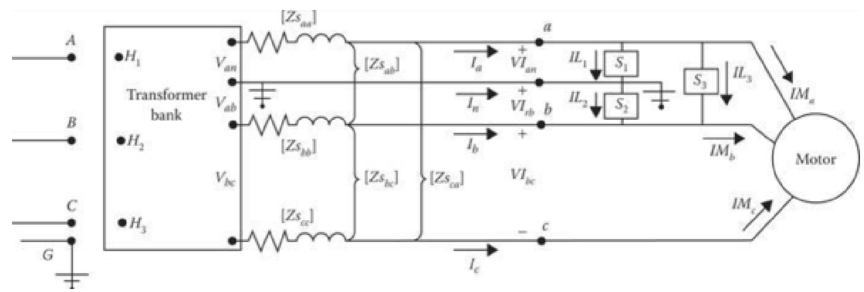
$$[V_{anbc}] = [V_{L_{anbc}}] + [Z_{s_{abc}}] \cdot [I_{abc}]$$

Modeling of Four Wire Secondary

- Eq. (40) is in the form of:

$$[V_{anbc}] = [a_s] \cdot [V_{L_{ankc}}] + [b_s] \cdot [I_{abc}] \qquad (41)$$

where,



$$[a_s] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[b_s] = \begin{bmatrix} Z_{s_{aa}} & Z_{s_{ab}} & Z_{s_{ac}} \\ -Z_{s_{ba}} & -Z_{s_{bb}} & -Z_{s_{bc}} \\ Z_{s_{ba}} - Z_{s_{ca}} & Z_{s_{bb}} - Z_{s_{bb}} & Z_{s_{bc}} - Z_{s_{cc}} \\ Z_{s_{ca}} - Z_{s_{aa}} & Z_{s_{cb}} - Z_{s_{bb}} & Z_{s_{sc}} - Z_{s_{ac}} \end{bmatrix}$$

- Because there are no shunt devices between the transformer and the loads, the currents leaving the transformers are equal to the line currents serving the loads. Therefore:

$$[d_s] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- The matrices for the forward sweep are:

$$[A_s] = [a_s]$$
$$[B_s] = [b_s]$$

Thank You!